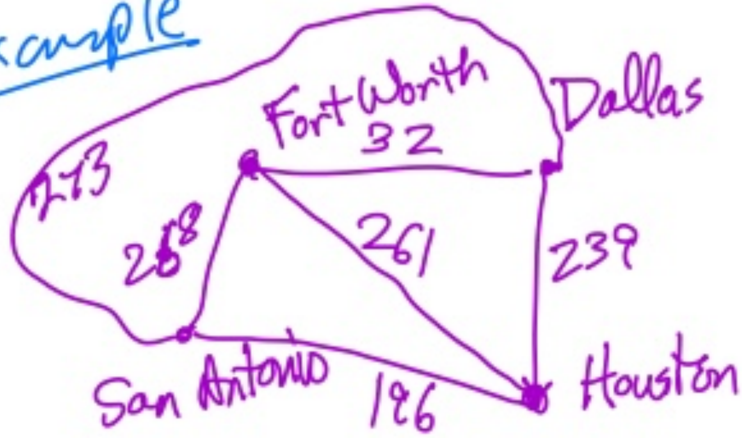
Example



Task: Find the shortest circuit in order to visit each city and come back to the starting point.

Possible answer: D H SA FW

725 =

725 = D → FW → SA → H

Observations:
- Starting point does not matter.
- Reversing the direction gives same result.
- How would we prove that is the shortest?

→ Write down every possible circuit, and compute the length of each.

- If it was true that the smallest 4 edges made a circuit, that would definitely be the shortest distance.

This is called the traveling salesman problem.
What is unsolved?

- A fast algorithm that works all the time.

If $n = \#$ of nodes, does the algorithm take $S(n)$ steps where S is no larger than a polynomial in n.

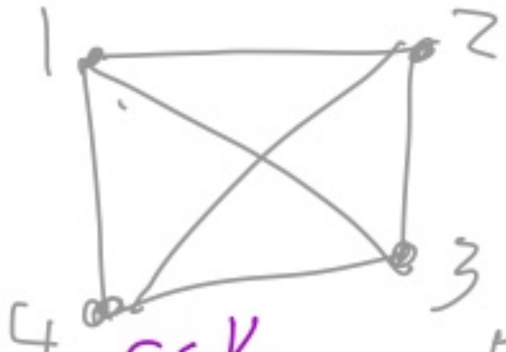
eg $|S(n)| \leq 1000\, n^{200}$

[It is believed to be greater than polynomial growth (eg. exponentially increasing in time as n increases).]

Given a Graph $G = (V, E)$

V = set of vertices

E = set of edges
- directed: ordered pairs of vertices
- undirected: sets of 2 vertices.



$G = K_4$

$G = (V, E)$

$V = \{1, 2, 3, 4\}$

$E = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}\}$

$|V|$ = number of elements in the set V
"cardinality of the set" $\quad |V| = 4$

$|E| = 6$

For $n \geq 1$ $\quad K_n$ = complete graph on n vertices.
every possible edge connecting 2 vertices is included in E.

$\Rightarrow$ in $K_n \Rightarrow |V| = n$

K_5 

$$|E| = \sum_{j=1}^{n-1} j = \frac{n(n-1)}{2}$$

$(n-1) + (n-2) + \cdots + 2 + 1$

$K_3 =$ 

$= \left(\begin{array}{l}\text{\# of ways to choose 2} \\ \text{vertices from a set of } n \text{ vertices}\end{array}\right)$

$${}_nC_2 = \binom{n}{2} = \frac{n!}{(n-2)!\,2!} = \frac{n(n-1)}{2}.$$

Given a graph G and a vertex v in the graph, the degree $\deg(v)$ of the vertex v is the number of edges coming out of v
$=$ (# of vertices connected by an edge to v) $+ 2$(# of loop).

$\leftarrow \deg(a) = 5, \deg(b) = 3$
$\deg(c) = 3, \deg(d) =$

Question: In K_n (complete graph with n vertices),

what is the degree of each vertex?

Answer: $n-1$

In a directed graph, each vertex v has an out-degree $\deg^+(v)$ and an in-degree $\deg^-(v)$

$\deg^+(v) =$ # of edges going out from v

$\deg^-(v) =$ # of edges coming in to v.

Note $\deg^+(v) + \deg^-(v) = \deg(v)$

ignoring directions.

Note: We can have graphs where vertices can have a degree of zero.

eg $G = (V, E)$

$V = \{1, 2, 3, 4\}$

$E = \{\{1,2\}, \{1,3\}\}$

$\deg(4) = 0$ ← 4 is called an isolated vertex.